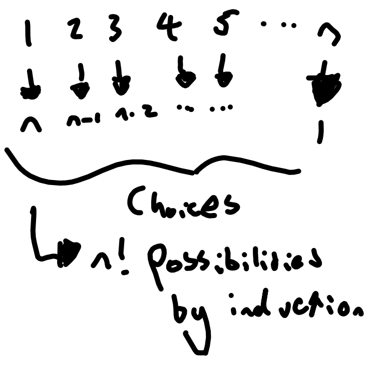
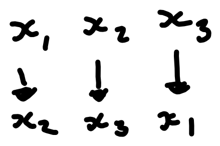
Lecture One

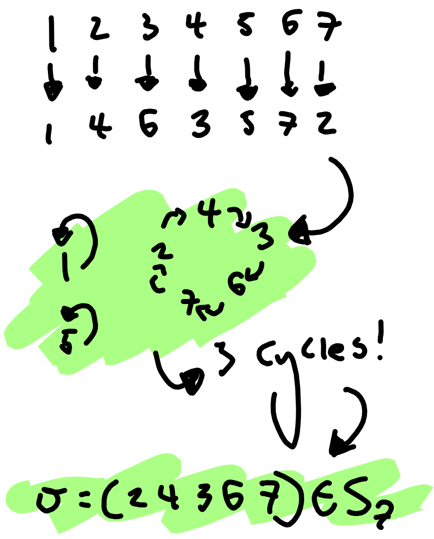
Have to internalise theories before using them - group theories from the start.

- A subsection/topic is one page of lecture notes - three a week. There are then handouts with more examples and proofs, and exercises to be completed in practical sessions /after lectures before the following week.
- **I forgot how much this guy yaps* - don't do daily games before his lectures, do them during >:)*
  - IT HAS BEEN TWENTY MINUTES WITH NO CONTENT AHHHHHHHHH
  - Nevermind love this guy, *waggle waggle*
  - Just realised Dan does way more work when it's his supervisor doing the lecture; wild.
- **Assessment Weighting:** Portfolio (at-home/in-class assessments 1:3 split) 40% / Final Exam 60%
- 1.1. Introduction to Permutations

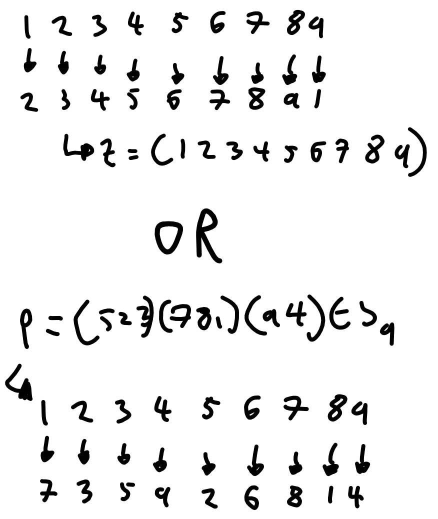




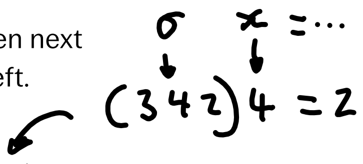
  - **Permutation**: re-arranging a set; a bijection (injective & surjective, one-to-one & onto). Just a complete unique map of one set to another set.
  - The set (group) of all permutations of a set, X, is Sym(x). If X is just {1,2,3,...,n}, we write S_n by convention instead.
  - The modulus of these sets (S_n), |S_n|, is n! - initially n choices, but then just n-1 choices moving an element down, then n-2 choices through the next, etc.
  - For finite x, |Sym(x)|=|x|! - proven inductively like on the left, for the previous argument.



  - Considering a specific example of the former, S_7, for example an permutation sigma within this set (see to the right) we can find a three distinct cycles.
  - These permutations always decompose into cycles, which can simplify our notation a bit: (1)(2 4 3 6 7)(5) - the three cycles with their elements, in order, and can be read as something repeatable within a bracket.



  - By convention, we exclude one-cycles, like the aforementioned (1) or (5) - single element cycles.
  - Hence for our example, sigma would be (2 4 3 6 7) in S_7 and excluded numbers are "fixed" (one cycles).
  - *Whenever he says six-seven I get naam flashbacks*
  - Another more complex example (see left) exemplifies the usefulness of the new notation.



  - NOTE: within permutations it's left -> right, but when they're written next to each other then you operate BETWEEN permutations right -> left. (maybe?? idk he keeps saying he'll go back to it, adhd ahh guy)
  - **NOTATION NOTE**: We write sigma x instead of sigma(x) to separate it from the bijection notation.
  - **FORMAL PERMUTATION DEFINITION**: Let a_1, a_2, ..., a_r be distinct elements of a set X. Then sigma = (a_1, a_2, ..., a_r) is the permutation of X sending a_1 -> a_2, a_2->a_3, ..., a_r->a_1, and fixing everything in X | {x_1,a_2,...,a_r}. This sigma is called a **cycle of length r**.
  - *These notes are gonna be annoying to sort, he's so all over the place grr*
  - **Pairwise Disjoint Definition**: A collection of cycles in Sym(x) is disjoint (NOTHING IN COMMON) if no element in X appears in two or more of them.
  - For example, cycles (1 2 3) and (4 7 5) are disjoint, but not (1 2 3) and (4 3 7 5).
  - **Uh oh controversial remark (APPARENTLY VERY IMPORTANT):** (1 2 3) and (3 1 2) are identical. YEAH DUH NOT VERY CONTROVERSIAL - just start with lowest num by convention to make comparison easier.
- 1.2. Products of Permutations

  - **Definition**: Let sigma and rho be permutations of a set X.
    The product sigma rho means (see right).
    Essentially, work right -> left, sort of like functions: sin cos 30 -> sin(cos 30).
  - This definition is by convention which conflicts between academics - **look into this (!!)**
  - *"it's definitely easier to see more do it because I'll waffle" - yeah we know bucko*
  - oh it's just like sigma(rho(1 2 3 4 5 ... N)) where N is from S_N, work within cycles too to make it easier
  - like sigma=(1 2 3) and rho = (1 5 3 4) in S_6, given sigma rho, becomes:
    sigma rho 1 = sigma 5 = 5
    sigma rho 5 = sigma 3 = 1
    sigma rho 1 = sigma 5 = 5 CYCLE: (1 5)
  - Then keep going for the next cycle(s), now starting at 2...
    sigma rho 2 = 3
    sigma rho 3 = 4
    sigma rho 4 = 2
    sigma rho 2 = 3 CYCLE: (2 3 4)
  - Then the last one is (6), fixed, so the final answer is: sigma rho = (1 5)(2 3 4)(6).
  - Like composing functions - light work.
  - *ew I just realised he writes tau really weird*
  - Another example, sigma = (1 2), rho = (2 3), tau = (3 5)...
    sigma rho tau 1 = (1 2)(2 3)(3 5) 1 = 2 - feeding the number through the cycles right -> left
    sigma rho tau 2 = (1 2)(2 3)(3 5) 2 = 3
    sigma rho tau 3 = (1 2)(2 3)(3 5) 3 = 5
    sigma rho tau 5 = (1 2)(2 3)(3 5) 5 = 1 CYCLE: (1 2 3 5)
  - Hence (1 2 3 5)(4).
  - YET ANOTHER EXAMPLE: (3 5)(2 4 6 8)(2 3 4) = ?
    (1)(2 5 3 6 8)(4)(7) - directly found by plugging in numbers in mah head.
  - Is kinda like simplifying permuations?
  - YIPPEEEEE IT'S OVER >:)))))))