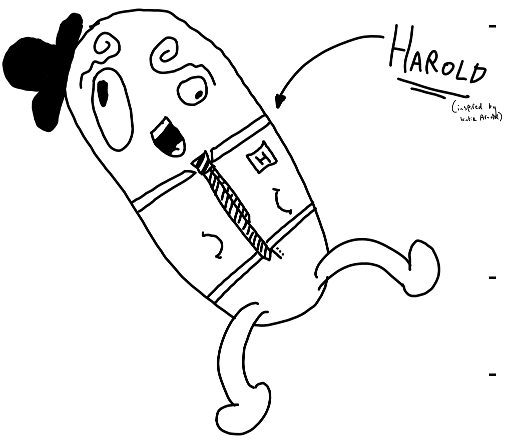
Lecture 2 - Theory & Practical

- **Commutativity**: Permutation multiplication is not commutative, i.e., xy != yx. Essentially like the composition of matrices, where each permutation is a function/bijective mapping.
  Exception: Disjoint cycles ARE commutative, since they don't affect each other at all.
- **Inverses**: Apply inverses to the right, as with our new normal convention right -> left order of operations, e.g. sigma rho = tau phi -> sigma = tau phi rho inverse.
- **Freaky Proposition (recap)**: If sigma & rho are disjoint then sigma rho = rho sigma, otherwise not equal.
- *My mcdon is getting cold, pls hurry up to questions Mr. Simon :(*
- Section 1.3. Properties of Permutations
  - **FREAKIER Proposition**: Products of permutations are associative; just composition of functions.
    $\forall \sigma, \rho, \tau \in Sym(X),$
    $(\sigma\rho)\tau = \sigma(\rho\tau)$
  - Identity permuation, e (or 1, Id, 0, I, among others), in Sym(X) is the bijection that sends x to x for all x in X; self-mapping, e.g. in S_n, e = (1)(2)(3)...(n) (or empty set if excluding one-cycles like normal).
  - **SUPER MEGA FREAKY Proposition:** For all sigma and rho in Sym(X):
    1. sigma e = e sigma = sigma *(identity element)*
    2. sigma rho in Sym(X) *(closure)*
    3. there exists inverse sigma in Sym(X) such that sigma inverse sigma = e = sigma sigma inverse *(inverse element)*
    just normal group proving shennanigans



  - **UBER SUPER MEGA FREAKY Proposition:** Let X be a finite set. Every permutation in Sym(X) can be written as a product of disjoint cycles.
  - **DIDDY Proposition**: Suppose sigma in SYm(X). We know we can write it as a product of disjoint cycles: sigma = c1 c2 .. cn. Since, the inverse of any cycle (x1 x2 ... xm) is (xm ... x2 x1), then, sigma inverse = c1 inverse c2 inverse ... cn inverse.
    Also called the boring ahh "quick inverse theorem", lame
  - The order of g in Sym(X) is the smallest n in Sym(X) that ... (idk I dozed off, probably something to do with inverses)
- 2.1. Groups (recap)
  - A group is a set G with an operation * on G such that...
    1. **Closure**: for any two elements, g, h, in the group, g * h in G.
    2. **Identity**: there exists an element e_G in G such that for all g in G, e_G*g=g=g*e_G
    I WANNA RED BULL SO BADDDDDD NOM NOM NOM NOM I'M TWEAKING
    3. **Inverse**: for all g in G there exists g inverse in G such that g*g inverse = e = g inverse * g
    I wonder if Diddy every went to Epstein's island - nvm alex broke my heart
    4. **Associative**: for all g,h,k in G blah blah blah you already know this big boss man
  - Notice this allows groups to be written as permutations! Yippee!!
  - **Ooooo so important example oooo thinks it's sooooo coool**: If X is a set then Sym(X) is a group, under the multiplication of permutations. RELATED TO KATIE'S THEOREM (KATIE ARNOLD!!!!) actually like cayley problem, that guy did too much
  - *yawn* wait I thought we were doing a practical today :'(
  - if G & H are two groups then their direct product/cartesian product is a group denoted G x H = {(g,h):g in G, h in H)} & (g1, h1) * (g2, h2) = (g1*g2,h1*h2) omg who'd have thunk it
  - WAIT WE ARE IN JUNCTION LATER UGHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH not a happy bunny
  - **Abelian groups** - just commutative innit omg lil bro hurry up with the recap AND WHAT DO YOU MEAN WE HAVEN'T COVERED CYCLIC GROUPS WE DID THAT DONKEY YEARS AGO
  - bro accidentally finished, nvm love this guy, just like me fr, hate it when I accidentally finish :((in alex or dan, sometimes tom when he cums over) must've been the wind
  - nevermind bro hit us with **order is number of elements in a group** like no shizzle lil brizzle WAIT AND IT'S DENOTED |G| NEVERMIND SO WORTH COMING TO THIS LECTURE
  - "nothing exciting now" yeah no shizzle my drizzle
  - PRACTICAL TIME LET'S GOOOOOOO MCDON AND RED BUL

Practical time bishes

Q 1.43. d/e/f (fun qs?)
I'm stuck stepbro !!
"let your friends help"
OKAY!! !!

1.41A.
$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 7 & 2 & 1 & 5 & 6 & 3 \end{array} \to (1\ 4)(2\ 7\ 3)(5)(6)$$
$(14)(273)$

B. $(1\ 3)(2\ 5)$

C.
$$\begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 11 & 8 & 10 & 4 & 3 & 2 & 6 & 1 & 5 & 9 & 12 \end{array}$$

2. A. $\underbrace{(5\ 7\ 9)(1\ 3\ 2)}_{k}\underbrace{(2\ 3\ 1)(9\ 7\ 5)}_{k^{-1}}$