1.4 Exercises 1- Exercises on permutations

Question 1.4.1. (a) Write the following permutation as a product of disjoint cycles:

$$
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
4 & 7 & 2 & 1 & 5 & 6 & 3
\end{array}
$$

(b) Write the following permutation as a product of disjoint cycles:

$$
\begin{array}{ccccc}
1 & 2 & 3 & 4 & 5 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
3 & 5 & 1 & 4 & 2
\end{array}
$$

(c) Write the following permutation out in full (using the above arrow notation):

$$(1\ 7\ 2\ 11\ 9)(4\ 10\ 5)(3\ 8\ 6) \in S_{12}$$

Question 1.4.2. Let $g = (1\ 4\ 2\ 7\ 8)$, $h = (1\ 4\ 5)$ and $k = (5\ 7\ 9)(1\ 3\ 2)$. Write the following as a product of disjoint cycles.

(a) k^{-1} (Check your answer by calculating kk^{-1})

(b) ghk

(c) $k^{-1}ghk$

Question 1.4.3. The *order* of a permutation σ is written $o(\sigma)$, and is defined to be the smallest natural number $n \geq 1$ such that $\sigma^n = e$. E.g. $o((1\ 2\ 3)) = 3$ and $o(e) = 1$. We will discuss the order of group elements in more detail later in the course, but for now try to answer the following questions.

(a) What is the order of the permutation $(1\ 2\ 3\ 4\ 5)$?

(b) What is the order of the permutation $(1\ 5\ 7)(2\ 3\ 6)$?

(c) What is the order of the permutation $(1\ 3\ 5)(2\ 4)$?

(d) Find an element of S_{10} with order 15.

(e) Is there an element of S_{10} with order 19? If so, find it; if not, why not?

(f) Suppose a permutation $g \in S_n$ is written as a product of disjoint r_i-cycles,

$$g = c_1 c_2 \cdots c_m.$$

Can you find a formula to calculate $o(g)$? Can you prove your formula is correct?

[Hint: first try to work out a formula for the order of a cycle of length r_i.]

Question 1.4.4. Find a permutation $\sigma \in S_4$ that satisfies the following permutation equation:

$$(1\ 3\ 2)\sigma = (1\ 2)(3\ 4)$$

Question 1.4.5. Find a permutation $\sigma \in S_7$ that satisfies the following permutation equation:

$$(1\ 3\ 2)(5\ 7\ 4)\sigma = (1\ 2)(3\ 4)$$

Question 1.4.6. Find a permutation $\sigma \in S_9$ that satisfies the following permutation equation:

$$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9)\sigma = (9\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1)$$