

11.4 Exercises 11- Exercises on abelian groups

Notation: if G has a subgroup that is isomorphic to a group H , then we write $H \lesssim G$. (S)

Question 11.4.1. Let G and H be groups. Prove that $G \times H \cong H \times G$.

Question 11.4.2. Let G and H be groups, with $K \leq G$ and $J \leq H$. Prove that

$$K \times J \leq G \times H.$$

Question 11.4.3. Let p be a prime and let $r, s \geq 0$ be integers. Prove that if $s \leq r$, then

$$C_{p^s} \lesssim C_{p^r}$$

(i.e. the group C_{p^r} has a subgroup that is isomorphic to C_{p^s}).

Question 11.4.4. Find, up to isomorphism, all abelian groups of order 72.

Question 11.4.5. Is the following true? Give a proof or counterexample. If G is a finite abelian group of order m , and $k|m$, then G has a subgroup of order k .

Hint. (Question 0.1.1) Try the map $\theta : G \times H \rightarrow H \times G$ given by $\theta((g, h)) = (h, g)$.

Hint. (Question 0.1.2) Use the quick subgroup test.

Hint. (Question 0.1.3) Try to find an element $h \in C_{p^r}$ that has order p^s . Then $H = \langle h \rangle \cong C_{p^s}$.

Hint. (Question 0.1.4) Use the Fundamental Theorem of Finite Abelian Groups.

Hint. (Question 0.1.5) This is a tricky question! Use The Fundamental Theorem of Finite Abelian Groups and the statements in Questions 0.1.2 and 0.1.3.