2.5 Exercises 2- Exercises on groups

(S)

Question 2.5.1. Let G be a group with $g, h \in G$. Prove that, for all $n \in \mathbb{N}$,

$$(g^{-1}hg)^n = g^{-1}h^n g.$$

Question 2.5.2. [Warning! Really easy proofs — don't overthink this.]
Let G be a group and $H \leq G$.

(a) Prove that $|H| = 1$ if and only if $H = \langle e_G \rangle$.

(b) Suppose $|G|$ is finite. Prove that $|H| = |G|$ if and only if $H = G$.

(c) Is the statement in (b) true if $|G|$ is infinite?

Question 2.5.3. Let G be a group with $H \leq G$ and $K \leq G$. Prove that $H \cap K \leq G$.

Question 2.5.4. Let G be a finite group and $g \in G$. Prove that $|\langle g \rangle| = o(g)$.

Question 2.5.5. Let $g \in S_{10}$ be the permutation $g = (1\ 3\ 5\ 7)(2\ 4)$. Write down all the elements in the following.

(a) $\langle g \rangle$

(b) $\langle g^2 \rangle$

Question 2.5.6. Show that for all $n \in \mathbb{N}$, the group $\mathbb{Z}_n$ is cyclic, with $\mathbb{Z}_n = \langle [1]_n \rangle$.