Tensor Analysis 25-26 - Portfolio - Part A

Instructions

(i) This list is worth 25% of the Portfolio marks.

(ii) The questions **are not organised by difficulty**.

(iii) Submit your solutions on Blackboard together with the part B of this assessment.

(iv) **Submit your work** (Part A+Part B) as an attachment to sepssubmissions@lincoln.ac.uk as well.

(v) Write your solutions on paper sheets, then scan them and submit through Turnitin. If you have no scanner available, you may also take photos on your phone, but note that you may have to bundle those together in a **single file for uploading**. Writing your solutions on a tablet is also allowed.

(vi) **Justify all your answers and write down all the steps of your calculations.**

(vii) Submit only one version of each solution (cross out wrong attempts).

Question A.1

[25 marks]

(a) Use suffix notation to simplify the following vector expression and remove all cross products. [10 marks]

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})\mathbf{a},$$

where $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ are vectors in $\mathbb{R}^3$.

(b) In this question, T_i denotes a first-rank tensor expressed in an orthogonal coordinate system, whose components are all non-zero.

(i) Show that the transformation rule for the derivative $\frac{\partial T_i}{\partial x^k}$ is [6 marks]

$$\left(\frac{\partial T_i}{\partial x^k}\right)' = \frac{\partial^2 x^m}{\partial x'^i \partial x'^k} T_m + L_{im} L_{kn} \frac{\partial T_m}{\partial x^n},$$

where $L_{im} = \frac{\partial x^m}{\partial x'^i}$ is the transformation matrix.

(ii) Write down the transformation rule that $\frac{\partial T_i}{\partial x^k}$ **would** have to satisfy in order to be a tensor. [4 marks] Then explain, using the result from part (i), why $\frac{\partial T_i}{\partial x^k}$ is **not** a tensor.

[You may use the result of part (i) even if you have not shown it.]

(iii) Show that the quantity [5 marks]

$$A_{ik} = \frac{\partial T_i}{\partial x^k} - \frac{\partial T_k}{\partial x^i}$$

transforms as a second-rank tensor.

[You may use the result stated in part (i), even if you have not shown it.]

Important:

- You must **justify every step** in your working.
- You can use results or formulas from the lectures, but you **must state them clearly**.