Tensor Analysis – Practical 1

Information:

- Please make sure to complete **all** exercises **before** the next lecture.
- The exercises marked with [**See lecture**] were solved in class.
- The exercises are **not organised by difficulty**.

1.1 Write in suffix notation:

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}.$$

1.2 [**See Lecture**] Consider the 3×3 matrix $\mathbf{C}$ given by the product $\mathbf{C} = \mathbf{AB}$, where

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}.$$

Verify that $C_{ij} = A_{ik}B_{kj}$.

1.3 Let $\mathbf{A}$ and $\mathbf{B}$ be the 3×3 matrices

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}.$$

Show, using suffix notation, that

$$\mathbf{AB} \neq \mathbf{BA},$$

i.e. matrix mutiplication does not commute.

1.4 [**See Lecture**] Let $\mathbf{A}$ and $\mathbf{B}$ be the 3×3 matrices

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}.$$

Show, using suffix notation, that

$$(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T,$$

where A^T is the transpose of A.

1.5 Let L, M and N be three 3×3 matrices. Show, using suffix notation, that

$$(LMN)^T = N^T M^T L^T.$$

1.6 [**See Lecture**] Simplify the suffix notation expression $\delta_{ij}a_j b_{\ell ck}\delta_{i\ell}$ and write the result in vector form.

1.7 Recall the alternating tensor ϵ_{ijk}. Evaluate the following in vector notation.

(1) ϵ_{122},

(2) ϵ_{321},

(3) $\epsilon_{223} + \epsilon_{111}$.

1.8 [**See Lecture**] Use suffix notation to show that $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$.

1.9 [**See Lecture**] Write the vector equation

$$\mathbf{a} \times \mathbf{b} + (\mathbf{a} \cdot \mathbf{d})\mathbf{c} = \mathbf{e}$$

in suffix notation.