Tensor Analysis – Practical 10

Information:

- Please make sure to complete **all** exercises **before** the next lecture.
- The exercises marked with [**See lecture**] were solved in class.
- The exercises are **not organised by difficulty**.

10.1 Given a 2-dimensional metric tensor

$$[g_{ij}] = \begin{pmatrix} 1 & 0 \\ 0 & e^{2x} \end{pmatrix}$$

for the a coordinate system $(x^1, x^2) = (x, y)$, calculate the components of the Ricci tensor R_{ij}.

10.2 Consider the 3-dimensional space in spherical coordinates that has arc length element

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

Compute the Ricci tensor components R_{kk} for this coordinate system. Here $(x^1, x^2, x^3) = (r, \theta, \phi)$

10.3 Given a 3-dimensional metric tensor

$$g_{ij} = \begin{pmatrix} e^{2u} & 0 & 0 \\ 0 & e^{2v} & 0 \\ 0 & 0 & e^{2w} \end{pmatrix}$$

in coordinates $(x^1, x^2, x^3) = (u, v, w)$. Find the Ricci tensor components R_{ij}.

Revision exercises

10.4 Consider the coordinate system having coordinates $(x^1,\ x^2, x^3) = (\phi,\ \theta, z)$ and position vector

$$\mathbf{r} = \cos\phi(\sin\theta + \cos\theta)\mathbf{i}_1 + \sin\phi(\sin\theta + \cos\theta)\mathbf{i}_2 + z\mathbf{i}_3,$$

where $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$ are the usual Cartesian basis vectors.

(1) Find basis vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_2$.

(2) Show that this is an orthogonal coordinate system.

10.5 Again, consider the coordinate system of Question 10.4.

Find the covariant components of the vector

$$\mathbf{A} = -\mathbf{i}_1 + \mathbf{i}_2$$

with respect to the basis $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ of Question 10.4.

10.6 Compute the metric coefficients of the arc length and the components of the covariant metric tensor g_{ii} for the coordinate system of Question 10.4.

10.7 Determine all Christoffel symbols of the first and of the second kind for the coordinate system of Question 10.4.

10.8 Determine the following Riemann-Christoffel tensors for the coordinate system of Question 10.4.

$$R^1{}_{122}, \quad \text{and} \quad R^r{}_{ijk}.$$