Tensor Analysis – Practical 7

Information:

- Please make sure to complete **all** exercises **before** the next lecture.
- The exercises marked with [**See lecture**] were solved in class.
- The exercises are **not organised by difficulty**.

7.1 Compute the outer product following pairs of tensors.

(1) $M = [M_{ij}] = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$ and $N = [N_{ij}] = \begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix}$.

(2) $P = [P_i] = \begin{pmatrix} 0 & 1 \end{pmatrix}$ and

$$Q = [Q_{ijkl}] = \begin{pmatrix} Q_{1111} & Q_{1112} & Q_{1211} & Q_{1212} \\ Q_{1121} & Q_{1122} & Q_{1221} & Q_{1222} \\ Q_{2111} & Q_{2112} & Q_{2211} & Q_{2212} \\ Q_{2121} & Q_{2122} & Q_{2221} & Q_{2222} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 1 & 1 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 3 \end{pmatrix}.$$

(3) $R = [R_i] = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$ and $S = [S_{ij}] = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & -7 \\ 0 & 1 & 0 \end{pmatrix}$.

7.2 Let T_{ij} and $R_{ijk\ell}$ be tensors of rank 2 and 4, respectively. Show that

$$T_{km}R_{kpqm}$$

is a rank 2 tensor.

7.3 Given that

$$[R_{ik}] = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 1 & 3 & 4 \end{pmatrix}, \qquad \mathbf{A} = \mathbf{i}_1 + 2\mathbf{i}_2 - \mathbf{i}_3.$$

find the inner products $R_{ik}A_i$ and $R_{ik}A_k$.

7.4 Let T_{ik} and $\mathbf{A}$ be the same as in the preceding problem, and let

$$\mathbf{B} = 4\mathbf{i}_1 + 5\mathbf{i}_2 + 3\mathbf{i}_3.$$

Find the inner product $T_{ik}A_iB_k$.

7.5 Find all possible contractions for the following dimension-two tensors.

(1) $[A_{ij}] = \begin{pmatrix} 1 & -1 \\ 4 & 0 \end{pmatrix}$.

(2)

$$[B_{ijk\ell}] = \begin{pmatrix} B_{1111} & B_{1112} & B_{1211} & B_{1212} \\ B_{1121} & B_{1122} & B_{1221} & B_{1222} \\ B_{2111} & B_{2112} & B_{2211} & B_{2212} \\ B_{2121} & B_{2122} & B_{2221} & B_{2222} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 7 \\ 4 & 0 & 2 & 11 \\ 2 & 2 & -1 & -3 \\ 1 & 128 & 2 & 0 \end{pmatrix}.$$