1.1. $\cos\theta = \dfrac{\underline{a}\cdot\underline{b}}{|\underline{a}||\underline{b}|}$

$= \dfrac{a_i b_i}{\sqrt{a_j a_j}\sqrt{b_k b_k}}$

1.2. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

$\hookrightarrow C = AB$

$\therefore C_{ij} = A_{i1}B_{1j} + A_{i2}B_{2j} + \dots + A_{in}B_{nj}$

$= \sum_{k=1}^{n} A_{ik}B_{kj}$

$= A_{ik}B_{kj}$ in Suffix notation. □

* 1.3. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

$\hookrightarrow AB = A_{i1}B_{1j} + A_{i2}B_{2j} + \dots + A_{in}B_{nj}$

$= \sum_{k=1}^{n} A_{ik}B_{kj}$

$= A_{ik}B_{kj}$

$BA = B_{i1}A_{1j} + B_{i2}A_{2j} + \dots + B_{in}A_{nj}$

$= \sum_{k=1}^{n} B_{ik}A_{kj}$

$= B_{ik}A_{kj}$

$= A_{kj}B_{ik}$

$\hookrightarrow AB = A_{ik}B_{kj} \neq A_{kj}B_{ik} = BA \because ik \neq kj,\ kj \neq ik$ □

1.4. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

$\hookrightarrow (AB)^T = ((AB)^T)_{ij}$

$= (AB)_{ji}$

$= A_{jk}B_{ki}$

$B^T A^T = (B^T A^T)_{ij}$

$= (B^T)_{ik}(A^T)_{kj}$

$= B_{ki}A_{jk}$

$= A_{jk}B_{ki}$

$\hookrightarrow (AB)^T = A_{jk}B_{ki} = B^T A^T$ □

1.5. Let L, M, N be 3x3 Matrices.

$\hookrightarrow (LMN)^T = ((L_{ik}M_{ki})N_{ij})^T$

$= (L_{ik}M_{kl}N_{lj})^T$

$= L_{jk}M_{kl}N_{lj}$

$N^T M^T L^T = (N^T)_{ij}(M^T)_{ij}(L^T)_{ij}$

$= ((N^T)_{ik}(M^T)_{kj})(L^T)_{ij}$

$= (N^T)_{ik}(M^T)_{kl}(L^T)_{lj}$

$= N_{ki}M_{lk}L_{jl}$

$= L_{jl}M_{lk}N_{ki}$

$= L_{jk}M_{kl}N_{li}$ by swapping dummy indices

$\hookrightarrow (LMN)^T = L_{jk}M_{kl}N_{lj} = N^T M^T L^T$ □

1.6. $\delta_{ij}a_j b_l c_k \delta_{il} = a_i b_i c_k$

$= (\underline{a}\cdot\underline{b})\underline{c}$

1.7. For ϵ_{ijk},

1. $\epsilon_{122} = 0$
2. $\epsilon_{321} = -1$
3. $\epsilon_{222} + \epsilon_{\dots} = 0 + 0 = 0$

1.8. $LHS = \underline{a} \times \underline{b} = (\underline{a}\times\underline{b})_i$

$= \epsilon_{ijk}a_j b_k$

$RHS = -\underline{b}\times\underline{a} = -(\underline{b}\times\underline{a})_i$

$= -\epsilon_{ijk}b_j a_k$

$= -\epsilon_{ikj}a_j b_k$

$= \epsilon_{ijk}a_j b_k$

$\therefore LHS = RHS$ □

1.9. $\underline{a}\times\underline{b} + (\underline{a}\cdot\underline{d})\underline{c} = \underline{e}$

$\hookrightarrow (\underline{a}\times\underline{b})_i + (\underline{a}\cdot\underline{d})_j \underline{c}_i = \underline{e}_i$

$\hookrightarrow \epsilon_{ijk}a_j b_k + a_j d_j c_i = e_i$