2.1.1. Curl Curl, Curl grad, div grad ✓

V→V V→V, V→V S→V, V→S S→V

2\. Div grad, div Curl, div div

V→S S→V, V→S V→V, V→S V→S ϟ

3\. Grad div, Curl grad, Curl Curl, grad grad, div Curl

S→V V→S, V→V S→V, V→V V→V, S→V S→V ϟ, V→S V→V

4\. Div grad, div Curl, Curl grad, div Curl ✓

S V V S, V V V S, S V V V, V V V S

2.2. $\delta_{ij} c_j + \epsilon_{kji} a_k b_j = d_\ell e_m c_i b_\ell c_m$

$\to c_i + \underbrace{\epsilon_{ikj} a_k b_j}_{\underline{a} \times \underline{b}} = c_i \underbrace{b_\ell d_\ell}_{b \cdot d} \underbrace{c_m e_m}_{c \cdot}$

$\to \underline{c} + (\underline{a} \times \underline{b}) = \underline{c}(b \cdot d)(c \cdot e)$

2.3. $(\underline{u} \times (\underline{v} \times \underline{w})) \cdot \underline{z}$

$\to (\underline{u} \times (\underline{v} \times \underline{w}))_i z_i$

$\to (\epsilon_{ijk} u_j (\underline{v} \times \underline{w})_k) z_i$

$\to (\epsilon_{ijk} u_j (\epsilon_{k\ell m} v_\ell w_m)) z_i$

$\to \epsilon_{ijk} \epsilon_{k\ell m} u_j v_\ell w_m z_i$

$\to (\delta_{i\ell}\delta_{jm} - \delta_{im}\delta_{j\ell}) u_j v_\ell w_m z_i$

$\to u_j v_i w_j z_i - u_j v_j w_i z_i$

$\to (\underline{u} \cdot \underline{w})(\underline{v} \cdot \underline{z}) - (\underline{u} \cdot \underline{v})(\underline{w} \cdot \underline{z})$

2.4. $[\underline{u} \times (\nabla \times \underline{v})]_i = \epsilon_{ijk} u_j (\nabla \times \underline{v})_k$

$= \epsilon_{ijk} u_j \epsilon_{k\ell m} \frac{\delta v_m}{\delta x_\ell}$

$= (\delta_{i\ell}\delta_{jm} - \delta_{im}\delta_{j\ell}) u_j \frac{\delta v_m}{\delta x_\ell}$

$= u_j \frac{\delta v_j}{\delta x_i} - u_j \frac{\delta v_i}{\delta x_j}$ ①

$\underline{v} \leftrightarrow \underline{u}$

Hence, $[\underline{u} \times (\nabla \times \underline{v}) + \underline{v} \times (\nabla \times \underline{u})]_i = u_j \frac{\delta v_j}{\delta x_i} - u_j \frac{\delta v_i}{\delta x_j} + v_j \frac{\delta u_j}{\delta x_i} - v_j \frac{\delta u_i}{\delta x_j}$ ②

$= u_j \frac{\delta v_j}{\delta x_i} + v_j \frac{\delta u_j}{\delta x_i} - (\underline{v}(\nabla \cdot \underline{u}) + \underline{u}(\nabla \cdot \underline{v}))_i$

2.

$= [\nabla(\underline{u} \cdot \underline{v})]_i - (\underline{v}(\nabla \cdot \underline{u}) + \underline{u}(\nabla \cdot \underline{v}))_i$

$\to \nabla(\underline{u} \cdot \underline{v}) = \underline{u} \times (\nabla \times \underline{v}) + \underline{v} \times (\nabla \times \underline{u}) + \underline{v}(\nabla \cdot \underline{u}) + \underline{u}(\nabla \cdot \underline{v})$ □

2.5.1. $\epsilon_{123}\epsilon_{312} \stackrel{?}{=} \delta_{11}\delta_{22} - \delta_{12}\delta_{21}$

$\to$ LHS $= 1 \cdot 1 = 1$,

& RHS $= 1 \cdot 1 - 0 \cdot 0 = 1$.

$\therefore$ LHS = RHS □

2\. $\epsilon_{213}\epsilon_{321} \stackrel{?}{=} \delta_{22}\delta_{11} - \delta_{21}\delta_{12}$

$\to$ LHS $= -1 \cdot -1 = 1$,

& RHS $= 1 \cdot 1 - 0 \cdot 0 = 1$.

$\therefore$ LHS = RHS □

2.6.1. $\delta_{ij}\epsilon_{ijk} = \epsilon_{iik} = \underline{0}$

2\. $\epsilon_{ijk}\epsilon_{i\ell m} = \epsilon_{jki}\epsilon_{i\ell m} = \delta_{j\ell}\delta_{km} - \delta_{jm}\delta_{k\ell}$

3\. $\epsilon_{ijk}\epsilon_{ijm} = \epsilon_{jki}\epsilon_{ijm} = \delta_{jj}\delta_{km} - \delta_{jm}\delta_{kj} = 3\delta_{km} - \delta_{km} = 2\delta_{km}$

4\. $\epsilon_{ijk}\epsilon_{ijk} = \epsilon_{jki}\epsilon_{ijk} = \delta_{jj}\delta_{kk} - \delta_{jk}\delta_{kj} = 9 - \delta_{jj} = 9 - 3 = 6$

2.7.1. $\epsilon_{pqr}\epsilon_{ijk} M_{pi} M_{qj} M_{rk}$

$= \epsilon_{pqr}\epsilon_{pqr}|M|$

$= \epsilon_{qrp}\epsilon_{pqr}|M|$

$= (\delta_{qq}\delta_{rr} - \delta_{qr}\delta_{rq})|M|$

$= (9 - \delta_{qq})|M|$

$= 6|M|$ □

2\. … } relabelling

3\. … }

2.8. $[\nabla f(r)]_i = \frac{\delta f(r)}{\delta x_i}$

$= \frac{\delta f(r)}{\delta r}\frac{\delta r}{\delta x_i}$

$= f'(r)\frac{\delta}{\delta x_i}(x_j x_j)^{\frac{1}{2}}$

$= \cdots$

WIP